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T.B.C. : B-UETC-O-PDV

Test Booklet Series

A

TEST BOOKLET

MATHEMATICS

Paper – III

Time Allowed : Two Hours ***Maximum Marks : 200***

INSTRUCTIONS

1. IMMEDIATELY AFTER THE COMMENCEMENT OF THE EXAMINATION, YOU SHOULD CHECK THAT THIS TEST BOOKLET ***DOES NOT*** HAVE ANY UNPRINTED OR TORN OR MISSING PAGES OR ITEMS, ETC. IF SO, GET IT REPLACED BY A COMPLETE TEST BOOKLET.
2. **Please note that it is the candidate's responsibility to encode and fill in the Roll Number and Test Booklet Series Code A, B, C or D carefully and without any omission or discrepancy at the appropriate places in the OMR Answer Sheet. Any omission/discrepancy will render the Answer Sheet liable for rejection.**
3. You have to enter your Roll Number on the Test Booklet in the Box provided alongside. ***DO NOT*** write ***anything else*** on the Test Booklet.
4. This Test Booklet contains **100** items (questions). Each item comprises four responses (answers). You will select the response which you want to mark on the Answer Sheet. In case you feel that there is more than one correct response, mark the response which you consider the best. In any case, choose ***ONLY ONE*** response for each item.
5. You have to mark all your responses ***ONLY*** on the separate Answer Sheet provided. See directions in the Answer Sheet.
6. ***All*** items carry equal marks.
7. Before you proceed to mark in the Answer Sheet the response to various items in the Test Booklet, you have to fill in some particulars in the Answer Sheet as per instructions sent to you with your Admission Certificate.
8. After you have completed filling in all your responses on the Answer Sheet and the examination has concluded, you should hand over to the Invigilator ***only the Answer Sheet***. You are permitted to take away with you the Test Booklet.
9. Sheets for rough work are appended in the Test Booklet at the end.
10. **Penalty for wrong answers :**

    THERE WILL BE PENALTY FOR WRONG ANSWERS MARKED BY A CANDIDATE IN THE OBJECTIVE TYPE QUESTION PAPERS.

    (i) There are four alternatives for the answer to every question. For each question for which a wrong answer has been given by the candidate, **one-third** of the marks assigned to that question will be deducted as penalty.

    (ii) If a candidate gives more than one answer, it will be treated as a **wrong answer** even if one of the given answers happens to be correct and there will be same penalty as above to that question.

    (iii) If a question is left blank, i.e., no answer is given by the candidate, there will be **no penalty** for that question.

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1. Let S be any set and P(S) be its power set. We define a relation R on P(S) by ARB to mean $A \subseteq B$ for all $A, B \in P(S)$. Consider the following in respect of the relation R :

1. R is a reflexive relation.
2. R is an anti-symmetric relation.
3. R is a symmetric relation.
4. R is a transitive relation.

Which of the above are correct ?

(a) 1, 3 and 4

(b) 3 and 4 only

(c) 1, 2 and 4

(d) 1 and 2 only

2. What is the real part of $(\sin x + i \cos x)^5$, where $i = \sqrt{-1}$?

(a) $-\cos 5x$

(b) $-\sin 5x$

(c) $\cos 5x$

(d) $\sin 5x$

3. If 5^{99} is divided by 13, then the remainder is

(a) 1

(b) 5

(c) 8

(d) 11

4. The number of consecutive odd integers whose sum can be expressed as $50^2 - 13^2$ is

(a) 33

(b) 35

(c) 37

(d) 39

5. A group of order 4 is

(a) always cyclic

(b) always non-abelian

(c) abelian and may not be cyclic

(d) always non-cyclic

6. If a and b are rational and $(b^2 + 1)$ is not a perfect square, then the quadratic equation with rational coefficients whose one root is $\frac{a}{2}\left(b+\sqrt{1+b^2}\right)$ is

(a) $x^2 - 2abx - a^2 = 0$

(b) $4x^2 - 4abx - a^2 = 0$

(c) $x^2 - abx - a^2 = 0$

(d) $x^2 - abx + a^2 = 0$

7. If $2|z-1| = |z-2|$ and $3(x^2 + y^2) = kx$, then what is k equal to ?

(a) 2/3

(b) 4/3

(c) 4

(d) 1

8. Let $a_1, a_2, a_3, ...$ be a sequence of real numbers such that $|a_i| = |a_{i-1} + 1|$ for $i \geq 2$ and $a_1 = 0$. If A denotes the arithmetic mean of $a_1, a_2, a_3, ..., a_n$ then which one of the following is correct ?

(a) $2nA = a_{n+1}^2 - n$

(b) $2nA = a_n^2 - n$

(c) $2nA = a_{n+1}^2 - n - 1$

(d) $2nA = a_n^2 - n - 1$

9. If A is a non-singular matrix of order 3, then what is adj(adj A) equal to ?

(a) $|A|^3 A$

(b) $|A|^2 A$

(c) $|A|\, A$

(d) A

10. If A, B and C are the angles of an isosceles triangle, then what is

$$\begin{vmatrix} 1 & 2 & 1 \\ 1+\sin A & 2+\sin A+\sin B & 1+\sin C \\ \sin A(1+\sin A) & \sin A(1+\sin A)+\sin B\,(1+\sin B) & \sin C\,(1+\sin C) \end{vmatrix}$$

equal to ?

(a) 0

(b) 1

(c) $\sin A \cdot \sin B \cdot \sin C$

(d) None of the above

11. Let A and B be two 3×3 matrices whose determinants are 2 and 4 respectively. What is $\det(\operatorname{adj}(A^{-1}B))$ equal to ?

(a) $|A|$

(b) $|B|$

(c) $4|A|$

(d) $4|B|$

12. Let S be the set S = {2, 4, 6, 8, ..., 20}. Define the operation $p \odot_n q$ as remainder when pq is divided by n. Then the inverse of the element 2 in $(S, \odot_{22})$ is

(a) 12

(b) 8

(c) 6

(d) 4

13. If $|z - 25i| \le 15$ where $i = \sqrt{-1}$, then what is $|\max \text{amp}(z) - \min \text{amp}(z)|$ equal to ?

(a) $\cos^{-1}\left(\frac{3}{5}\right)$

(b) $\pi - 2\cos^{-1}\left(\frac{3}{5}\right)$

(c) $\frac{\pi}{2} + \cos^{-1}\left(\frac{3}{5}\right)$

(d) $\sin^{-1}\left(\frac{3}{5}\right) - \cos^{-1}\left(\frac{3}{5}\right)$

14. If the quadratic equation

$$x^2 - 4px + 4p(p-1) = 0,$$

where p is real, has its real roots greater than p, then p lies in the interval

(a) $(4, \infty)$

(b) $(-\infty, -4)$

(c) $(-4, 0)$

(d) $(-4, -1)$

15. What is the sum of the first 10 terms of the series $\frac{2}{3} + \frac{5}{3^2} + \frac{8}{3^3} + \ldots$?

(a) $1 + \frac{3}{4}\left(1 - \frac{1}{3^9}\right) - \frac{29}{2 \times 3^{10}}$

(b) $1 + \frac{3}{4}\left(1 - \frac{1}{3^8}\right) - \frac{25}{2 \times 3^9}$

(c) $1 + \frac{3}{4}\left(1 - \frac{1}{3^9}\right) - \frac{25}{2 \times 3^{10}}$

(d) $1 + \frac{3}{4}\left(1 - \frac{1}{3^9}\right) - \frac{29}{3^{10}}$

16. A square matrix of third order is said to be skew-symmetric if

(a) All elements of leading diagonal are zero

(b) $a_{ij} = a_{ji}$

(c) All elements of leading diagonal are 1

(d) $a_{ij} = -a_{ji}$

where a_{ij} being element in the i^{th} row and j^{th} column.

17. The equations

$kx + y + z = k - 1, \quad x + ky + z = k + 1,$

$x + y + kz = k - 1$ has no solution if

(a) $k = 1$ only

(b) $k \neq -2$

(c) $k = -2$ or 1

(d) $k = -2$ only

18. What is the value of

$$\left(\frac{i + \sqrt{3}}{-i + \sqrt{3}}\right)^{52722} + \left(\frac{i - \sqrt{3}}{i + \sqrt{3}}\right)^{40305}$$

where $i = \sqrt{-1}$?

(a) $\sqrt{2}$

(b) $\sqrt{3}$

(c) 2

(d) 3

19. The function

$$f(x) = a_0 + a_1 |x| + a_2 |x|^2 + a_3 |x|^3$$

is differentiable at $x = 0$

(a) only when $a_1 = 0$

(b) only when $a_1 = a_3 = 0$

(c) only when $a_1 = a_2 = a_3 = 0$

(d) for any values of a_0, a_1, a_2 and a_3

20. If the complex numbers z_1, z_2, z_3 are in AP, they lie on

(a) a circle

(b) a line

(c) a parabola

(d) an ellipse

21. Which one of the following binary operations $*$ is associative on the set of real numbers ?

(a) $a * b = a^b$

(b) $a * b = a + b - 1$

(c) $a * b = \frac{a}{b}, \; b \neq 0$

(d) $a * b = a - b$

22. All the fourth roots of unity are

(a) 1, 1, −1, −1

(b) $i, i, -i, -i$

(c) $1, -1, i, -i$

(d) $-i, -i, -i, -i$

where $i = \sqrt{-1}$.

23. Consider the following in respect of the equation $(x + 2)^2 - 3 |x + 2| + 2 = 0$:

1. The sum of all possible roots of the equation is -8.

2. The product of all possible roots of the equation is 0.

Which of the above statements is/are correct ?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

24. Three straight lines l_1, l_2, l_3 are parallel and lie on the same plane. 5 points are taken on line l_1, 6 points are taken on line l_2 and 7 points are taken on line l_3. What is the maximum number of triangles formed with vertices at these points ?

(a) 620

(b) 746

(c) 751

(d) 781

25. Consider the following statements in respect of the expansion $\frac{(1+x)^{2n}}{x^n}$:

1. Independent term does not exist in the expansion.

2. The coefficient of x is equal to coefficient of x^{-1} in the expansion.

Which of the above statements is/are correct ?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

26. If $A \in (0, 2\pi) - \{\pi\}$, how many solutions of $\cot\frac{A}{2} - \tan\frac{A}{2} = 2$ are possible ?

(a) Only one

(b) Two

(c) Four

(d) No solution is possible

27. Consider the following statements :

1. $\sin 75° + \cos 105° \neq \cos\theta$ for any θ, where $0 < \theta < 60°$.

2. $\sin\theta + \cos\theta < 1$ for all θ, where $90° < \theta < 120°$.

Which of the above statements is/are correct ?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

28. Let $T = \{\theta \in \mathbf{R} : 3\theta$ is not of the form $k\pi$ for any $k \in Z\} \cap [0, 2\pi]$.

Consider the following statements :

Statement-I :

There exists at least one $x \in \mathbf{R} \setminus (-1, 1)$ for which there exists no $t \in T$ such that $\frac{1 + 2\cos 2t}{\sin 3t} = x$.

Statement-II :

For any $\theta \in T$, $\frac{1 + 2\cos 2\theta}{\sin 3\theta} = \operatorname{cosec}\theta$.

Which one of the following is correct in respect of the above statements ?

(a) Both the statements are true and statement-II is the correct explanation of statement-I.

(b) Both the statements are true but statement-II is not the correct explanation of statement-I.

(c) Statement-I is true, but statement-II is false.

(d) Statement-I is false, but statement-II is true.

29. Consider the following statements :

Statement-I :

There exists no triangle ABC satisfying $\frac{\cos A}{a} = \frac{\cos B}{b} = \frac{\cos C}{c} = \frac{1}{2R}$, where R is the circum-radius of the triangle ABC.

Statement-II :

If ABC is an isosceles triangle satisfying $b^2 = c^2 + a^2$, then $\frac{a}{\cos A} = \frac{c}{\cos C} = b$.

Which one of the following is correct in respect of the above statements ?

(a) Both the statements are true and statement-II is the correct explanation of statement-I.

(b) Both the statements are true but statement-II is not the correct explanation of statement-I.

(c) Statement-I is true, but statement-II is false.

(d) Statement-I is false, but statement-II is true.

30. Let ABC be a triangle with $\angle B = 60°$.

Statement-I :

If $a = b \sin C + c \sin B$, then $\angle C \neq 45°$.

Statement-II :

$$b^2 (1 - \sin 2C) = c^2 \left(\frac{2-\sqrt{3}}{2}\right).$$

Which one of the following is correct in respect of the above statements ?

(a) Both the statements are true and statement-II is the correct explanation of statement-I.

(b) Both the statements are true but statement-II is not the correct explanation of statement-I.

(c) Statement-I is true, but statement-II is false.

(d) Statement-I is false, but statement-II is true.

31. It is given that $(\sin^{-1} x) \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi^2}{36}$.

Which one of the following is ***not*** correct ?

(a) $\sin^{-1} x - \cos^{-1}\left(\frac{1}{2}\right) \neq 0$

(b) $\sin^{-1} x + \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$

(c) $\sin^{-1} x = \frac{1}{5} \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

(d) $(\sin^{-1} x)^2 = \frac{1}{9}\left[\cos^{-1}\left(\frac{1}{2}\right)\right]^2$

32. Consider the following statements :

1. If α, β are supplementary angles and $\cot(\alpha - \beta) = 1$, then $\tan 2\alpha = \cot 2\alpha$.

2. If α, β are complementary angles and $\tan(\alpha - \beta) = 1$, then $\sec 2\beta = \operatorname{cosec} 2\beta$.

Which of the above statements is/are correct ?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

33. Consider the following statements :

1. If α, β are acute angles and $\tan(\alpha + \beta) = 1$ and $\sqrt{3} \sec(\alpha - \beta) = 2$, then $\tan 2\alpha = \cot 15°$.

2. If α, β are the angles in the second quadrant and $\operatorname{cosec}(\alpha - \beta) = -\sec(\alpha + \beta) = 2$, then $\sin \beta = \cos 15°$.

Which of the above statements is/are correct ?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

34. What is the maximum value of

$$5 \cos \theta + 3 \cos \left(\theta + \frac{\pi}{3}\right) + 3 \ ?$$

(a) 11
(b) 10
(c) 5
(d) 1

35. Consider the following statements :

1. If $\theta = -\frac{17\pi}{4}$, then $\sin^8 \theta = \frac{1}{8}$.

2. If $\theta = \frac{231\pi}{6}$, then $\sin^6 3\theta = 1$.

Which of the above statements is/are correct ?

(a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2

36. For how many distinct values of A between 0° and 360° is the expression

$\frac{\sin A + \sin 2A + \sin 3A}{\cos A + \cos 2A + \cos 3A}$ undefined ?

(a) 2
(b) 4
(c) 6
(d) 8

37. In a triangle ABC if $\cot \frac{A}{2}$, $\tan \frac{B}{2}$, $\cot \frac{C}{2}$ are in HP, then what is the value of

$\tan \frac{A}{2} \tan \frac{C}{2}$?

(a) -1
(b) $\frac{1}{2}$
(c) 1
(d) 2

38. If the function $f(x) = \sin x + \cos (xa)$ is periodic, then 'a' is

(a) always a natural number
(b) always an integer
(c) an irrational number
(d) a rational number

39. At how many points do $y = x$ and $y = \tan x$ intersect ?

(a) Zero
(b) Only one
(c) Two
(d) Infinite

40. ABCDEFG is a 7-sided polygon which is not regular. If its angles are in AP, then which one of the following is correct ?

(a) Exactly three of its angles are greater than 125°.
(b) Exactly four of its angles are greater than or equal to the angle of a regular polygon of 7-sides.
(c) Exactly three of its angles are less than or equal to $\frac{5\pi}{7}$ radian.
(d) The sum of the greatest angle and the least angle is greater than $\frac{10\pi}{7}$ radian.

41. If $x = \varphi(t)$, $y = \psi(t)$, then what is $\frac{d^2y}{dx^2}$ equal to ?

(a) $\frac{\varphi'\psi'' - \psi'\varphi''}{(\varphi')^2}$

(b) $\frac{\varphi'\psi'' - \psi'\varphi''}{(\varphi')^3}$

(c) $\frac{\varphi''}{\psi''}$

(d) $\frac{\varphi'\psi'' + \psi'\varphi''}{(\varphi')^2}$

where dashes denote the derivative with respect to t.

42. Let $f(x) = \sin x$, $g(x) = x^2$ and $h(x) = \ln x$ be functions of real variable $x > 0$. Suppose $fog(x)$ means $f[g(x)]$. If $F(x) = [(hog)of](x)$, what is $F''(x)$ equal to ?

(a) $2 \operatorname{cosec}^2 x$

(b) $2 \sec^2 x$

(c) $-2 \operatorname{cosec}^2 x$

(d) None of the above

43. If $f(x) = a \ln |x| + bx^2 + x$ has its extreme values at $x = -1$ and $x = 2$, then what is the value of 'a' ?

(a) 1

(b) 2

(c) -1

(d) -2

44. If $g(x) = x^3$ and $3 f(x) = 4x^3 - 12x$ where $0 \le x \le 2$, then $g[f(x)]$ will attain its greatest value at

(a) $x = 2$

(b) $x = 0$

(c) $x = 1$

(d) $x = \frac{1}{3}$

45. If $5y = -3[x] + 4[\tan x] + 3|y|$ where [.] is the greatest integer function, then y as a function of x is

(a) not continuous at $x = 0$

(b) continuous at $x = 0$

(c) differentiable at $x = 0$

(d) continuous at $x = 0$ but not differentiable at $x = 0$

46. If $f(x) = \frac{x^3\sqrt{1+x^2}}{2-x}$, $0 \le x \le 1$ and

$f(x) = \frac{x^3\sqrt{1+x^2}}{2+x}$, $-1 \le x \le 0$, then what is

$\int_{-\frac{1}{2}}^{\frac{1}{2}} f(x)\,dx$ equal to ?

(a) 4

(b) $\frac{3}{2}$

(c) 1

(d) 0

47. Let F(x) be a twice differentiable function with $F''(x) = -F(x)$ and $F'(x) = G(x)$. If $H(x) = \{F(x)\}^2 + \{G(x)\}^2$ and $H(5) = 5$, then what is H(0) equal to ?

(a) 0

(b) 5

(c) 9

(d) 10

48. If $f(a) = 2$, $f'(a) = 1$, $g(a) = -1$, $g'(a) = 2$, then what is $\lim_{x \to a} \left[\frac{g(x)f(a) - g(a)f(x)}{x - a}\right]$ equal to ?

(a) -5

(b) $\frac{1}{5}$

(c) 5

(d) $-\frac{1}{5}$

49. The function $f(x) = e^x(1 - x^2)$ is

(a) increasing for $x > \sqrt{2}$

(b) decreasing for $x < \sqrt{2}$

(c) increasing for $|x - 1| < \sqrt{2}$

(d) increasing for $|x + 1| < \sqrt{2}$

50. Consider the function $f : [0, \pi] \to [0, 1]$ defined by $f(x) = \sin\left(\frac{x}{3}\right)$. The function f is

(a) one-one

(b) onto

(c) both one-one and onto

(d) neither one-one nor onto

51. What is $\int_0^{\sqrt{\pi}} x\, e^{x^2} \sin(x^2)\, dx$ equal to ?

(a) $\frac{e^\pi + 1}{2}$

(b) $\frac{e^\pi - 1}{4}$

(c) $\frac{e^\pi + 1}{4}$

(d) $e^\pi + 1$

52. If f(x) is a second order polynomial (or quadratic expression in x) and

$$\int_a^b f(x)\, dx = (a - b)(a^2 + b^2 + ab + 2),$$

then f(x) will be of the form

(a) $3x^2 + x + 2$

(b) $2x^2 - x$

(c) $-3x^2 - 2$

(d) $3x^2 + 2$

53. If $I_1 = \int_0^{\pi/2} \cos(\sin x)\, dx$, $I_2 = \int_0^{\pi/2} \sin(\cos x)\, dx$ and $I_3 = \int_0^{\pi/2} \cos x\, dx$, then which one of the following is correct ?

(a) $I_1 > I_2 > I_3$

(b) $I_3 > I_2 > I_1$

(c) $I_3 > I_1 > I_2$

(d) $I_1 > I_3 > I_2$

54. If $a > 1$, $b > 1$ then the minimum value of $\log_a b + \log_b a$ is

(a) 0

(b) 2

(c) 1

(d) None of the above

55. If $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$, then which of the following is/are correct ?

1. $\frac{dy}{dx} = \frac{2}{1+x^2}$ for $-1 < x < 1$

2. $\frac{dy}{dx} = -\frac{2}{1+x^2}$ for $x < -1$

3. $\frac{dy}{dx} = -\frac{2}{1+x^2}$ for $x > 1$

Select the correct answer using the code given below :

(a) 1 only

(b) 1 and 2 only

(c) 2 and 3 only

(d) 1, 2 and 3

56. The equation of the curve passing through the point (0, 1) and having x^3y^{-3} as the slope of the tangent to the curve at any point (x, y) is

(a) $x^4 - y^4 + 1 = 0$

(b) $x^4 + y^4 - 1 = 0$

(c) $x^3 + y^3 - 1 = 0$

(d) $x^3 - y^3 + 1 = 0$

57. If I_1 is the integrating factor of the differential equation $x\frac{dy}{dx} - y = x^2$ and I_2 is the integrating factor of the differential equation $x\frac{dy}{dx} + y = x^{-2}$, then which one of the following is ***not*** correct ?

(a) $I_1 I_2 = 1$

(b) $I_2 = x^2 I_1$

(c) $I_1 = x^2 I_2$

(d) $I_2 > I_1$ for $x > 1$

58. Consider the following differential equations :

1. $(x - y)\frac{dy}{dx} = 2x + y$

2. $x\cos\left(\frac{y}{x}\right)\frac{dy}{dx} = y\cos\left(\frac{y}{x}\right) + 4x$

3. $2x^2y^2\frac{dy}{dx} = x^2 + y^2$

4. $\sin x\frac{dy}{dx} = \cos x$

How many of the above are homogeneous ?

(a) One

(b) Two

(c) Three

(d) Four

59. If $f(x) = \frac{1-x}{1+x}$ where $x > 0$ and $x \neq 1$, then $f[f(x)] + f\left[f\left(\frac{1}{x}\right)\right]$ is

(a) less than 2

(b) greater than 2

(c) greater than or equal to 2

(d) equal to 2

60. What is $\int_1^3 \frac{[x^2]\,dx}{[x^2 - 8x + 16] + [x^2]}$, where [.] denotes the greatest integer function, equal to ?

(a) 4

(b) 3

(c) 2

(d) 1

61. If $\int \frac{g''(x)\, g(x)\, dx}{\{g'(x)\}^2} = x + \text{constant}$, then the function g(x) will be of the form

(a) $ax^2 + b$

(b) $a\, e^{bx^2}$

(c) $a\, e^{-bx}$

(d) $a\, e^{bx}$

where a and b are non-zero constants.

62. What is the area bounded by the curves $y = ln\, x$ and $y = (ln\, x)^2$?

(a) $e - 1$

(b) $e - 2$

(c) $3 - e$

(d) e

For the next three (03) items that follow :

Consider the function

$$f(x) = \int_1^x \{2(t-1)(t-2)^3 + 3(t-1)^2(t-2)^2\}\, dt.$$

63. The function attains local maximum at

(a) $x = 0$

(b) $x = 1$

(c) $x = 2$

(d) $x = 4$

64. What is the local maximum value of the function ?

(a) 0

(b) 1

(c) 4

(d) 16

65. Consider the following statements :

1. The function attains local minimum value at $x = \frac{7}{5}$.

2. $x = 2$ is the point of inflexion.

Which of the above statements is/are correct ?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

For the next two (02) items that follow :

Consider $f(x) = -1 + |x - 1|, \; -1 \le x \le 3$ and

$g(x) = 2 - |x + 1|, \; -2 \le x \le 2.$

66. For $x \in (0, 1)$, fog(x) is equal to

(a) $x - 1$

(b) $1 - x$

(c) $x + 1$

(d) $-x - 1$

67. Consider the following statements :

1. For $x \in (-1, 1)$, fof(x) = x.

2. For $x \in (-1, 2)$, gog(x) = x.

Which of the above statements is/are correct ?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

For the next three (03) items that follow :

Let $f : \mathbb{R} \to \mathbb{R}$ be defined by

$$f(x) = \frac{3x^2}{2} + x^2 \sin\left(\frac{1}{x}\right) \text{ for } x \neq 0 \text{ and } f(0) = 0.$$

68. The function f(x) is

(a) continuous and differentiable at $x = 0$

(b) nowhere continuous over $\mathbb{R}$

(c) continuous at $x = 0$, but not differentiable at $x = 0$

(d) nowhere differentiable over $\mathbb{R}$

69. The function f(x) has

(a) local maximum at $x = 0$

(b) local minimum at $x = 0$ but it has no absolute minimum

(c) absolute minimum at $x = 0$

(d) absolute maximum at $x = 0$

70. Let $x_1 = \frac{1}{2n\pi}$ and $x_2 = \frac{2}{(4n+1)\pi}$ where $n \in N$.

The derivative of the function f(x) attains

(a) positive value at x_1 and negative value at x_2

(b) positive value at x_1 and positive value at x_2

(c) negative value at x_1 and positive value at x_2

(d) negative value at x_1 and negative value at x_2

71. How many integral points are there within the graph of $|x| + |y| < 3$?

(a) 13

(b) 15

(c) 21

(d) 24

72. The distance of the point (4, 5) from the straight line joining the points (1, 2) and (–2, 3) measured parallel to the line $x + y + 1 = 0$ is

(a) 4 units

(b) $4\sqrt{2}$ units

(c) 6 units

(d) $6\sqrt{2}$ units

73. A double ordinate of the parabola $y^2 = 4ax$ is of length 8a. What is the angle between the lines from the vertex to its ends ?

(a) 30°

(b) 45°

(c) 60°

(d) 90°

74. For how many values of k, the line $3x - 4y = k$ may touch the circle $x^2 + y^2 - 4x - 8y - 5 = 0$?

(a) 1

(b) 2

(c) 3

(d) None of the values of k

75. What is one of the angles between the straight lines

$(x \cos\alpha - y \sin\alpha)^2 = (x^2 + y^2)\sin^2\alpha$?

(a) α

(b) 2α

(c) 4α

(d) $\alpha/2$

76. A straight line passes through a fixed point (h, g). The locus of the foot of the perpendicular on it drawn from the origin is

(a) a straight line

(b) an ellipse

(c) a parabola

(d) a circle

77. If the three distinct points $(t_i, 2at_i + at_i^3)$ for $i = 1, 2, 3$ are collinear, then the sum of the abscissa of the points is

(a) -1

(b) 0

(c) 1

(d) 3

78. Let (a, b) and (c, d) be two points in a plane. Any point on the line joining these points has coordinates

(a) $(a + kc, b + kd)$

(b) $(ka + c, kb + d)$

(c) $((1 - k) a + kc, (1 - k) b + kd)$

(d) $(a + (1 - k) c, b + (1 - k) d)$

where k is any real number.

79. The equation

$$|\vec{r}|^2 + \vec{r}\,.(2\hat{i} + 4\hat{j} - 2\hat{k}) - 10 = 0$$

represents a sphere of radius

(a) 2 units

(b) 3 units

(c) 4 units

(d) 5 units

80. What is $\int \frac{(ax+b)\,dx}{|ax+b|}$, where $a \neq 0$, $x \neq -\frac{b}{a}$, equal to ?

(a) $\frac{(ax+b)}{a} + c$

(b) $(ax + b) + c$

(c) $|x| + c$

(d) None of the above

where c is the constant of integration.

For the next three (03) items that follow :

Consider a point A(–2, 3, 0) above the line PQ. The line PQ passes through P(–3, 5, 2) and makes equal angles with the coordinate axes.

81. What are the coordinates of the foot of the perpendicular from A on the line PQ ?

(a) (– 4, 4, 1)

(b) (4, 4, 1)

(c) (– 2, 2, 1)

(d) (2, 2, 1)

82. What are the direction ratios of the line perpendicular to the line PQ ?

(a) $< 2, 1, -1 >$

(b) $< -2, 1, 1 >$

(c) $< 4, 1, 1 >$

(d) $< 1, 1, 1 >$

83. What is the square of the perpendicular distance of the point A from the line PQ ?

(a) 4

(b) 5

(c) 6

(d) 9

For the next two (02) items that follow :

A variable plane $\frac{x}{3a} + \frac{y}{3b} + \frac{z}{3c} = 1$ at unit distance from the origin cuts the coordinate axes at A, B and C respectively. The centroid of the triangle ABC satisfies the equation $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = k^2$.

84. The centroid of the triangle is at

(a) $\left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right)$

(b) (a, b, c)

(c) (3a, 3b, 3c)

(d) $\left(\frac{a}{2}, \frac{b}{2}, \frac{c}{2}\right)$

85. The value of k is

(a) $\frac{1}{9}$

(b) $\frac{1}{3}$

(c) 3

(d) 9

86. If ABCDEF is a regular hexagon with $\overrightarrow{AB} = \vec{a}$ and $\overrightarrow{BC} = \vec{b}$, then what is $\overrightarrow{CE}$ equal to ?

(a) $\vec{b} - \vec{a}$

(b) $\vec{b} - 2\vec{a}$

(c) $2\vec{b} - \vec{a}$

(d) $\vec{b} + \vec{a}$

87. If (0, 1) and (1, 0) are mid-points of the sides of a right-angled triangle, then consider the following statements :

1. (0, 0) can be the orthocentre of the triangle.

2. (1, 1) can be the orthocentre of the triangle.

Which of the above statements is/are correct ?

(a) 1 only

(b) 2 only

(c) Either 1 or 2

(d) Neither 1 nor 2

For the next three (03) items that follow :

The vectors $\vec{b} = (\tan\alpha)\hat{i} - \hat{j} + 2\sqrt{\sin\left(\frac{\alpha}{2}\right)}\,\hat{k}$ and $\vec{c} = (\tan\alpha)\hat{i} + (\tan\alpha)\hat{j} - 3\sqrt{\operatorname{cosec}\left(\frac{\alpha}{2}\right)}\,\hat{k}$ are orthogonal and a vector $\vec{a} = \hat{i} + 3\hat{j} + (\sin 2\alpha)\hat{k}$ makes an obtuse angle with z-axis.

88. What is/are the permissible value(s) of $\tan\alpha$?

(a) -2 only

(b) 3 only

(c) Both -2 and 3

(d) Neither -2 nor 3

89. In which quadrant does α lie ?

(a) First quadrant

(b) Second quadrant

(c) Third quadrant

(d) Fourth quadrant

90. What is α equal to ?

(a) $(4n + 1)\pi \pm \tan^{-1} 2$

(b) $(4n + 2)\pi \pm \tan^{-1} 2$

(c) $(4n + 1)\pi - \tan^{-1} 2$; $(4n + 2)\pi - \tan^{-1} 2$

(d) None of the above

where n is an integer.

91. The numbers 1, 2, 3, 4, 5, 6, 7, 8 are arranged in a random order. The probability that the digits 1, 2, 3, 4 appear as neighbours in that order is

(a) $\frac{1}{2}$

(b) $\frac{1}{128}$

(c) $\frac{1}{256}$

(d) $\frac{1}{336}$

92. The average marks of 10 students in a class was 60 with a standard deviation 4, while the average marks of other 10 students was 40 with a standard deviation 6. If all the 20 students are taken together, their standard deviation will be

(a) 5·0

(b) 7·5

(c) 9·8

(d) 11·2

93. The two lines of regression of y on x and x on y are $5y + 4x = 37$ and $y + 5x = 20$ respectively. The correlation between x and y will be

(a) $\frac{2}{5}$

(b) $\frac{-2}{5}$

(c) $\frac{1}{5}$

(d) $\frac{-1}{5}$

94. Correlation between two variables X and Y is given to be 0·6. These variables are transformed to new variables $u = -2X + 3$ and $v = 5Y - 2$. What will be the correlation between u and v ?

(a) 0·6

(b) – 0·6

(c) 0·2

(d) Information is insufficient

95. If A and B are any two events with $P(A) = 0{\cdot}6$, $P(B) = 0{\cdot}3$ and $P(A \cap B) = 0{\cdot}2$, what will be $P(A^c \mid B^c)$, where A^c is the complementary event of A ?

(a) 3/7

(b) 4/7

(c) 1/3

(d) 2/3

96. A point is chosen at random inside a rectangle measuring 5 inches by 6 inches. What is the probability that the point chosen at random inside the rectangle is at least one inch from the edge ?

(a) 5/6

(b) 4/5

(c) 3/4

(d) 2/5

97. A box contains three types of seeds : 50% of type A; 20% of type B and rest of type C. It is known that 20% of A, 30% of B and 30% of C germinate. A seed is drawn randomly from the box. What is its probability to germinate ?

(a) 0·25

(b) 0·50

(c) 0·80

(d) 1

98. A box contains a fair coin and a two-headed coin B. A coin is selected at random from the box and tossed twice. If head comes both the times, the probability that it is by the two-headed coin is

(a) 1/4

(b) 1/2

(c) 4/5

(d) 5/8

99. Some urns contain 4 white and 6 black balls, while one urn contains 5 white and 5 black balls. One urn is chosen at random from these and 2 balls are drawn from it, and both are found to be black. The probability that 5 white and 3 black balls remain in the chosen urn is 1/7. The total number of urns is

(a) 4

(b) 5

(c) 6

(d) 7

100. n observations on a variable X are $X_i = A + iB$ for $i = 1, 2, 3, \ldots, n$ where A, B are real constants. The mean of the observations is

(a) $A + B\left(\frac{n+1}{2}\right)$

(b) $nA + B\left(\frac{n+1}{2}\right)$

(c) $A + Bn\left(\frac{n+1}{2}\right)$

(d) $A + B\left(\frac{n}{2}\right)$

SPACE FOR ROUGH WORK

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