SCRA Exam : 2014

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**T.B.C. : B–QAPY–N–NBU** **Test Booklet Series**

**Serial No.** 228937

**A**

# TEST BOOKLET

## MATHEMATICS

## Paper—III

**Time Allowed : Two Hours** **Maximum Marks : 200**

## INSTRUCTIONS

1. IMMEDIATELY AFTER THE COMMENCEMENT OF THE EXAMINATION, YOU SHOULD CHECK THAT THIS TEST BOOKLET *DOES NOT* HAVE ANY UNPRINTED OR TORN OR MISSING PAGES OR ITEMS, ETC. IF SO, GET IT REPLACED BY A COMPLETE TEST BOOKLET.
2. **Please note that it is the candidate's responsibility to encode and fill in the Roll Number and Test Booklet Series A, B, C or D carefully and without any omission or discrepancy at the appropriate places in the OMR Answer Sheet. Any omission/ discrepancy will render the Answer Sheet liable for rejection.**
3. You have to enter your Roll Number on the Test Booklet in the Box provided alongside. *DO NOT* write *anything else* on the Test Booklet.
4. This Test Booklet contains **100** items (questions). Each item comprises four responses (answers). You will select the response which you want to mark on the Answer Sheet. In case you feel that there is more than one correct response, mark the response which you consider the best. In any case, choose *ONLY ONE* response for each item.
5. You have to mark all your responses *ONLY* on the separate Answer Sheet provided. See directions in the Answer Sheet.
6. All items carry equal marks.
7. Before you proceed to mark in the Answer Sheet the response to various items in the Test Booklet, you have to fill in some particulars in the Answer Sheet as per instructions sent to you with your Admission Certificate.
8. After you have completed filling in all your responses on the Answer Sheet and the examination has concluded, you should hand over to the Invigilator *only the Answer Sheet*. You are permitted to take away with you the Test Booklet.
9. Sheets for rough work are appended in the Test Booklet at the end.
10. **Penalty for wrong answers :**
    THERE WILL BE PENALTY FOR WRONG ANSWERS MARKED BY A CANDIDATE IN THE OBJECTIVE TYPE QUESTION PAPERS.
    (i) There are four alternatives for the answer to every question. For each question for which a wrong answer has been given by the candidate, **one-third** of the marks assigned to that question will be deducted as penalty.
    (ii) If a candidate gives more than one answer, it will be treated as a **wrong answer** even if one of the given answers happens to be correct and there will be same penalty as above to that question.
    (iii) If a question is left blank, i.e., no answer is given by the candidate, there will be **no penalty** for that question.

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**1.** The function $f: R \to R$ defined by

$$f(x) = (x - a)(x - b)(x - c)$$

where $a, b, c \in R$, is

*(a)* not one-one but onto

*(b)* one-one but not onto

*(c)* both one-one and onto

*(d)* neither one-one nor onto

**2.** If $A = \{1, 2, 3, 4\}$, then which of the following is/are the function(s) from $A$ to itself?

I. $f_1 = \{(x, y) \mid x + y = 5\}$

II. $f_2 = \{(x, y) \mid y < x\}$

Select the correct answer using the code given below.

*(a)* I only

*(b)* II only

*(c)* Both I and II

*(d)* Neither I nor II

**3.** If $f: R \to R$ be given by

$$y = f(x) = (x + 1)^2 - 1$$

then $f(x)$ is invertible if

*(a)* $y \geq -1$

*(b)* $-2 \leq y < -1$

*(c)* $-3 \leq y < -2$

*(d)* None of the above

**4.** The complex numbers $z$ satisfying $z^2 + |z| = 0$ are

*(a)* $0, i, -i$

*(b)* $0, 1, i, -i$

*(c)* $0, 1, -1, i, -i$

*(d)* $0, -1$

**5.** If $z_1, z_2, z_3$ are complex numbers such that

$$|z_1| = |z_2| = |z_3| = \left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right| = 1$$

then what is $|z_1 + z_2 + z_3|$ equal to?

*(a)* Less than 1

*(b)* Lies between 1 and 3

*(c)* 1

*(d)* 3

**6.** If the lines $x + 2ay + a = 0$, $x + 3by + b = 0$ and $x + 4cy + c = 0$ are concurrent, then $a, b, c$ are in

*(a)* HP

*(b)* AP

*(c)* GP

*(d)* None of the above

**7.** Let $(\alpha, \beta)$, $(\beta, \gamma)$ and $(\gamma, \alpha)$ be the roots of the equations $x^2 + px + qr = 0$, $x^2 + qx + rp = 0$, $x^2 + rx + pq = 0$ respectively. Then the product of their common roots $(\alpha\beta\gamma)$ is equal to

*(a)* $pqr$

*(b)* $2pqr$

*(c)* $3pqr$

*(d)* $p^2q^2r^2$

**8.** The roots of the equation

$$qx^2 - px + (0{\cdot}5p - 0{\cdot}25q) = 0$$

when $p < q$, where $p$, $q$ are real numbers, are always

*(a)* irrational

*(b)* real

*(c)* complex

*(d)* rational

**9.** If $z$ is a complex number, then the common roots of the equations

$$z^{1985} + z^{100} + 1 = 0$$

$$z^3 + 2z^2 + 2z + 1 = 0$$

are

*(a)* $\omega, \omega^2$

*(b)* $1, \omega, \omega^2$

*(c)* $-1, \omega, \omega^2$

*(d)* $-\omega, -\omega^2$

**10.** The range of $\theta$ in the interval $(0, \pi)$ such that the points $(3, 5)$ and $(\sin\theta, \cos\theta)$ lie on the same side of the line $x + y - 1 = 0$, is

*(a)* $(0, \pi/4)$

*(b)* $(0, \pi/2)$

*(c)* $(\pi/2, \pi)$

*(d)* $(0, \pi)$

**11.** A line passing through the point $(2, 2)$ encloses an area $\lambda$ with the axes. The intercepts on the axes made by the line are given by the two roots of

*(a)* $x^2 + 2|\lambda|x + |\lambda| = 0$

*(b)* $x^2 - 2|\lambda|x + |\lambda| = 0$

*(c)* $x^2 + |\lambda|x + 2|\lambda| = 0$

*(d)* $x^2 - |\lambda|x + 2|\lambda| = 0$

**12.** The area bounded by the curve $y = 2x^4 - x^2$, the $x$-axis and the two ordinates corresponding to minimal of the function is

*(a)* $\frac{1}{40}$ square unit

*(b)* $\frac{7}{120}$ square unit

*(c)* $\frac{1}{24}$ square unit

*(d)* None of the above

**13.** If

$$a \leq 3\cos x + 5\sin\left(x - \frac{\pi}{6}\right) \leq b$$

holds good for all $x$, then $a$ and $b$ are respectively

*(a)* $-4, 4$

*(b)* $-\sqrt{19}, \sqrt{19}$

*(c)* $-\sqrt{29}, \sqrt{29}$

*(d)* $-8, 8$

**14.** In a triangle $ABC$

$$\sin A \sin B \sin C = \frac{3+\sqrt{3}}{8}$$

$$\cos A \cos B \cos C = \frac{\sqrt{3}-1}{8}$$

Then what is the value of $\tan A + \tan B + \tan C$?

*(a)* $\sqrt{3}(2 - \sqrt{3})$

*(b)* $2 + \sqrt{3}$

*(c)* $2 - \sqrt{3}$

*(d)* $\sqrt{3}(2 + \sqrt{3})$

**15.** The graph of the function

$$y = \cos x \cos(x+2) - \cos(x^2 + 1)$$

is a

*(a)* straight line passing through the point $(0, -\sin^2 1)$ and parallel to $x$-axis

*(b)* straight line passing through the origin

*(c)* parabola with vertex $(0, -\sin^2 1)$

*(d)* None of the above

**16.** Addition is not a binary operation on the set

*(a)* $N$ of natural numbers

*(b)* $\{x : x$ is a real number and $|x| = 1\}$

*(c)* $Q$ of rational numbers

*(d)* $R$ of real numbers

**17.** The locus of the point of intersection of the straight lines

$$\frac{x}{a} + \frac{y}{b} = \lambda \quad \text{and} \quad \frac{x}{a} - \frac{y}{b} = \frac{1}{\lambda}$$

where $\lambda$ is a variable, is

*(a)* a circle

*(b)* a parabola

*(c)* an ellipse

*(d)* a hyperbola

**18.** If the product of $n$ positive numbers is unity, then their sum is

*(a)* a positive integer

*(b)* divisible by $n$

*(c)* equal to $(n^2 + 1)/n$

*(d)* never less than $n$

**19.** The number of numbers between 1 and $10^{10}$, which contain the digit 1, is

*(a)* $10^{10} - 9^{10} - 1$

*(b)* $9^{10}$

*(c)* $10^{10} - 9^{10}$

*(d)* None of the above

**20.** If $a, b, c$ are any three consecutive terms in an AP, then the line $ax + by + c = 0$

*(a)* has a fixed direction

*(b)* passes through the origin $(c \neq 0)$

*(c)* always passes through a fixed point

*(d)* None of the above

**21.** A five-digit number divisible by 3 is to be formed using the numbers 0, 1, 2, 3, 4, 5 without repetition. The total number of ways in which this can be done is

*(a)* 216

*(b)* 240

*(c)* 600

*(d)* 3125

**22.** If $\alpha$, $\beta$ are the roots of the equation

$$ax^2 + 3x + 2 = 0 \ (a < 0)$$

then $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$ is greater than

*(a)* 1

*(b)* 2

*(c)* 3

*(d)* None of the above

**23.** The number of terms in the expansion of $(2x + 3y - 4z)^n$, where $n$ is a positive integer, is

*(a)* $n + 1$

*(b)* $(n + 1)(n + 2)/2$

*(c)* $n(n + 1)/2$

*(d)* $(n - 1)(n - 2)/2$

**24.** If $\sin(x - y)$, $\sin x$ and $\sin(x + y)$ are in HP, then $[\sin x \sec(y/2)]$ is equal to

*(a)* $\pm\sqrt{2}$

*(b)* $\pm 1$

*(c)* $\pm 3$

*(d)* $\pm 2$

**25.** If

$$\tan A = \frac{1 - \cos B}{\sin B}$$

then what is $\tan 2A$ equal to?

*(a)* $\tan B$

*(b)* $\tan 2B$

*(c)* $\sin B$

*(d)* $\cos B$

**26.** If an angle $\alpha$ is divided into two parts $A$ and $B$ such that

$$A - B = x \quad \text{and} \quad \frac{\tan A}{\tan B} = k$$

then what is $\sin x$ equal to?

*(a)* $\frac{k+1}{k-1}\sin\alpha$

*(b)* $\frac{k+1}{(k-1)\sin\alpha}$

*(c)* $\frac{k-1}{(k+1)\sin\alpha}$

*(d)* $\frac{k-1}{k+1}\sin\alpha$

**27.** What is the sum of the first 30 terms of the series $1\times2+2\times3+3\times4+\ldots$?

*(a)* 21010

*(b)* 8920

*(c)* 22100

*(d)* 9920

**28.** The total number of ways of selecting two numbers from the set {1, 2, 3, ..., 30}, so that their sum is divisible by 3, is

*(a)* 95

*(b)* 145

*(c)* 190

*(d)* None of the above

**29.** The sum of $n$ terms of the series

$$1+(1+x)+(1+x+x^2)+(1+x+x^2+x^3)+\ldots$$

where $x < 1$, is

*(a)* $\frac{1}{1-x}$

*(b)* $\frac{n}{1-x}$

*(c)* $\frac{n}{1-x}-\frac{n(1-x^n)}{(1-x)^2}$

*(d)* $\frac{n}{1-x}-\frac{n(1-x^{n+1})}{(1-x)^2}$

**30.** The 5th term from the end in the expansion of $\left(x-\frac{1}{x}\right)^{3n}$ in increasing power of $x$, is [$n$ is a positive integer]

*(a)* $x^{8-3n}$

*(b)* $x^{7-3n}$

*(c)* $x^{3n-4}$

*(d)* None of the above

**31.** Consider the following statements :

1. $\frac{1}{1-\sin A} > 2\sin A + \frac{1}{1+\sin A}$ where $0° < A < 90°$

2. $\frac{1}{1+\cos A} \leq 2 - \frac{1}{1-\cos A}$ where $0° < A < 90°$

Which of the above statements is/are correct?

*(a)* 1 only

*(b)* 2 only

*(c)* Both 1 and 2

*(d)* Neither 1 nor 2

**32.** Consider the following statements :

1. If $0 < \tan A < 1$, then

$$\frac{1}{1-\tan A} + \frac{\cot A}{\cot A - 1} = \frac{\cot A}{\cot A + 1} + \frac{1}{1+\tan A}$$

2. If $\tan A > 1$, then

$$\frac{1}{1-\tan A} + \frac{1}{1+\tan A} < 0$$

Which of the above statements is/are correct?

*(a)* 1 only

*(b)* 2 only

*(c)* Both 1 and 2

*(d)* Neither 1 nor 2

For the next **02 (two)** items that follow :

Consider $S = \sum_{r=2}^{n} \sin(r\alpha)$

**33.** What is $S$ if $(n+2)\alpha = 2\pi$?

*(a)* 0

*(b)* 1

*(c)* $\frac{1}{\sqrt{2}}$

*(d)* $\frac{1}{2}$

**34.** What is $S$ if $(n-1)\alpha = 2\pi$?

*(a)* 2

*(b)* 1

*(c)* $\frac{1}{2}$

*(d)* 0

For the next **02 (two)** items that follow :

Consider

$$\sin 5\theta = 5\sin\theta - 20\sin^3\theta + k\sin^5\theta$$

**35.** What is the value of $k$?

*(a)* 5

*(b)* 11

*(c)* 16

*(d)* −16

**36.** What is

$$40\sin^3\theta - 32\sin^5\theta - 10\sin\theta + 2\sin 5\theta$$

equal to?

*(a)* 0

*(b)* 1

*(c)* 2

*(d)* None of the above

For the next **02 (two)** items that follow :

Consider

$$f(x) = 2\tan^{-1} x + \sin^{-1}\left(\frac{2x}{1+x^2}\right),\ x > 1$$

**37.** What is $f(x)$ equal to?

*(a)* $\sec^{-1} x$

*(b)* $\text{cosec}^{-1} x$

*(c)* $\pi$

*(d)* $\frac{\pi}{2}$

**38.** What is $f(5)$ equal to?

*(a)* $5\pi$

*(b)* $\pi$

*(c)* $\frac{\pi}{2}$

*(d)* $2\pi$

For the next **02 (two)** items that follow :

Let

$$z = (\cos\theta + i\sin\theta)(\cos 2\theta + i\sin 2\theta)(\cos 3\theta + i\sin 3\theta)$$

where $\theta \in R$.

**39.** If $z$ is real, then which one of the following is correct?

*(a)* $\theta \in \left\{\frac{k\pi}{3} : k \text{ is an integer}\right\} \cup \left\{\frac{(2r+1)\pi}{6} : r \text{ is an integer}\right\}$

*(b)* $\theta \in \left\{\frac{k\pi}{3} : k \text{ is an integer}\right\}$ only

*(c)* $\theta \in \left\{\frac{k\pi}{2} : k \text{ is an integer}\right\}$

*(d)* None of the above

**40.** If $z$ is purely imaginary, then which one of the following is correct?

*(a)* $\theta \in \left\{\frac{(4k+1)\pi}{12} : k \text{ is an integer}\right\}$ only

*(b)* $\theta \in \left\{\frac{(2k+1)\pi}{12} : k \text{ is an integer}\right\}$

*(c)* $\theta \in \left\{\frac{k\pi}{12} : k \text{ is an integer}\right\}$

*(d)* None of the above

For the next **02 (two)** items that follow :

The $p$th, $q$th, $r$th terms of an HP are $a, b, c$ respectively.

**41.** What is $\begin{vmatrix} bc & ca & ab \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix}$ equal to?

*(a)* 0 *(b)* 1

*(c)* $abc$ *(d)* $(abc)^{-1}$

**42.** What is

$$\begin{vmatrix} b^2c^2 + c^2a^2 + a^2b^2 & pbc + qca + rab & bc + ca + ab \\ pbc + qca + rab & p^2 + q^2 + r^2 & p + q + r \\ bc + ca + ab & p + q + r & 3 \end{vmatrix}$$

equal to?

*(a)* $(abc)^{-2}$

*(b)* $(abc)^2$

*(c)* 1

*(d)* 0

For the next **02 (two)** items that follow :

Consider the system of equations

$$x + y + z = 1$$
$$x + 2y + 4z = k$$
$$x + 4y + 10z = k^2$$

**43.** What is/are the value(s) of $k$ which make(s) the system of equations to possess the solution?

*(a)* 0

*(b)* 1 or 2

*(c)* 3 or 4

*(d)* None of the above

**44.** Consider the following statements :

1. The system of equations can have infinite solutions for some value of $k$.

2. The system of equations can have unique solution for some value of $k$.

Which of the above statements is/are correct?

*(a)* 1 only

*(b)* 2 only

*(c)* Both 1 and 2

*(d)* Neither 1 nor 2

**45.** For what value(s) of $n \geq 1$, where $n$ is a natural number, $A^n - nA + nI = I$, where $I$ is the identity matrix and $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$?

*(a)* $n = 1$ only

*(b)* $n = 2$ only

*(c)* For all values of $n$

*(d)* None of the values of $n$

For the next **02 (two)** items that follow :

Consider the function

$$f(x) = \begin{cases} x & \text{when } x \text{ is rational} \\ 1-x & \text{when } x \text{ is irrational} \end{cases}$$

on the interval $I = [0, 1]$.

**46.** The function is continuous at

*(a)* $x = 0{\cdot}5$ only

*(b)* every point in $I$

*(c)* every rational point in $I$

*(d)* every irrational point in $I$

**47.** Consider the following statements :

1. $f(x)$ has its own inverse in $I$.
2. $f(x)$ is differentiable at $x = 0{\cdot}5$.

Which of the above statements is/are correct?

*(a)* 1 only

*(b)* 2 only

*(c)* Both 1 and 2

*(d)* Neither 1 nor 2

**48.** The function

$$f(x) = \frac{k\sin x + 2\cos x}{\sin x + \cos x}$$

is increasing for

*(a)* $k < 0$

*(b)* $0 < k < 1$

*(c)* $1 < k < 2$

*(d)* $k > 2$

For the next **02 (two)** items that follow :

Consider the function

$$f(x) = (x-2)^3 (x-1)^2$$

**49.** Consider the following statements :

1. The function is neither increasing nor decreasing in the interval $[1, 2]$.
2. The function has neither relative maximum nor relative minimum at $x = 2$.

Which of the above statements is/are correct?

*(a)* 1 only

*(b)* 2 only

*(c)* Both 1 and 2

*(d)* Neither 1 nor 2

**50.** Consider the following statements :

1. The function attains relative maximum at $x = 1$.
2. The function attains relative minimum at $x = \frac{7}{5}$.

Which of the above statements is/are correct?

*(a)* 1 only

*(b)* 2 only

*(c)* Both 1 and 2

*(d)* Neither 1 nor 2

For the next **02 (two)** items that follow :

Consider

$$a_n = \int_0^{\pi} \frac{\sin(2n-1)x}{\sin x} dx$$

where $n$ is a natural number.

**51.** What is $a_{100} - a_{99}$ equal to?

*(a)* 0 *(b)* 1

*(c)* $\frac{\pi}{2}$ *(d)* $\pi$

**52.** $a_1, a_2, a_3, \ldots, a_n$ are

*(a)* in AP only

*(b)* in GP only

*(c)* both in AP and GP

*(d)* neither in AP nor in GP

For the next **02 (two)** items that follow :

Let $f(x) = x - \ln|2x+1|$ be defined for

$$x \in \left(-100, \frac{1}{2}\right) - \left\{-\frac{1}{2}\right\}$$

**53.** The function $f(x)$ is monotonically decreasing in the interval

*(a)* $\left(-\frac{1}{2}, \frac{1}{2}\right)$

*(b)* $\left(-100, -\frac{1}{2}\right)$

*(c)* $\left(\frac{1}{2}, 100\right)$

*(d)* $\left(\frac{1}{2}, 1\right)$

**54.** The function $f(x)$ is monotonically increasing in the interval

*(a)* $\left(-\frac{1}{2}, \frac{1}{2}\right)$

*(b)* $\left(-\frac{1}{2}, 0\right)$

*(c)* $\left(0, \frac{1}{2}\right)$

*(d)* $\left(-100, -\frac{1}{2}\right)$

For the next **02 (two)** items that follow :

Let $f(x) = (1-x)^n$, where $n$ is a non-negative integer.

**55.** What is the coefficient of $x^n$ in $(1-x)^n$?

*(a)* $n$

*(b)* $-n$

*(c)* $(-1)^n$

*(d)* None of the above

**56.** What is

$$f(0) + f'(0) + \frac{f''(0)}{2!} + \ldots + \frac{f^n(0)}{n!}$$

equal to?

*(a)* $2^n$

*(b)* 0

*(c)* 1

*(d)* $-1$

For the next **02 (two)** items that follow :

Consider the ellipses $4x^2 + y^2 = 1$ and $x^2 + 4y^2 = 1$.

**57.** What is the area common to both the ellipses?

*(a)* $\tan^{-1} 2$ square units

*(b)* $2\tan^{-1} 2$ square units

*(c)* $4\tan^{-1} 2$ square units

*(d)* None of the above

**58.** What is the bounded area not common to both the ellipses?

*(a)* $(\pi - \tan^{-1} 2)$ square units

*(b)* $(2\pi - \tan^{-1} 2)$ square units

*(c)* $(\pi - 2\tan^{-1} 2)$ square units

*(d)* None of the above

For the next **02 (two)** items that follow :

Consider the functions $f(x) = x^2$, $g(x) = 2x + 1$ and $h(x) = x - \frac{1}{2}$ on the interval $I = [0, 1]$.

**59.** Consider the following statements :

1. The function $(fg)(x)$ is always increasing on $I$.
2. The function $(fh)(x)$ is always increasing on $I$.

Which of the statements given above is/are correct?

*(a)* 1 only

*(b)* 2 only

*(c)* Both 1 and 2

*(d)* Neither 1 nor 2

**60.** Consider the following statements :

1. The function $(gh)(x)$ is always increasing on $I$.
2. The function $(f + g)(x)$ is always increasing on $I$.

Which of the above statements is/are correct?

*(a)* 1 only

*(b)* 2 only

*(c)* Both 1 and 2

*(d)* Neither 1 nor 2

**61.** If

$$I_1 = \int \frac{e^x dx}{(1+x^2)^2},\ I_2 = \int \frac{xe^x dx}{(1+x^2)^2},\ I_3 = \int \frac{x^2 e^x dx}{(1+x^2)^2}$$

then what is $I_1 - 2I_2 + I_3$ equal to?

*(a)* $\frac{e^x}{1+x^2} + c$

*(b)* $\frac{e^x}{(1+x^2)^2} + c$

*(c)* $\frac{2e^x}{1+x^2} + c$

*(d)* None of the above

where $c$ is the constant of integration.

**62.** What is

$$\sum_{r=1}^{n} \int_0^{\pi/2} (r + \sin\theta)^2 \cos\theta \, d\theta$$

equal to?

*(a)* $\frac{n(n^2+3n+3)}{3}$

*(b)* $\frac{n(n+1)(n+2)}{3}$

*(c)* $\frac{n(n+1)(2n+1)}{3}$

*(d)* $\frac{(n+1)(n+2)}{3}$

**63.** What is

$$\lim_{x \to 1} \frac{\sqrt{1-\cos(2x-2)}}{x-1}$$

equal to?

*(a)* $\sqrt{2}$

*(b)* $-\sqrt{2}$

*(c)* 0

*(d)* Limit does not exist

**64.** If

$$I = \int_0^{\pi/2} \frac{\cos x \, dx}{1+\cos x + \sin x}$$

then what is

$$\int_0^{\pi/2} \frac{dx}{1+\cos x + \sin x}$$

equal to?

*(a)* $\frac{I}{2}$

*(b)* $I$

*(c)* $\frac{\pi}{2} - 2I$

*(d)* None of the above

**65.** What is

$$\int_{-1}^{1} \frac{x \, dx}{x^4 + x^2 + 1}$$

equal to?

*(a)* 0

*(b)* 1

*(c)* 2

*(d)* None of the above

**66.** The differential equation

$$y\frac{dy}{dx} + x = a$$

where $a$ is a constant, represents

*(a)* a set of circles having centre on the $y$-axis

*(b)* a set of parabolas

*(c)* a set of circles having centre on the $x$-axis

*(d)* a set of straight lines

**67.** What is the degree of the differential equation

$$\left(\frac{d^3y}{dx^3}\right)^{2/3} + 4 - 3\frac{d^2y}{dx^2} + 5\frac{dy}{dx} = 0\,?$$

*(a)* 1 *(b)* 2

*(c)* 3 *(d)* $\frac{2}{3}$

**68.** Consider the following statements :

1. $\int_0^a f(x)\,dx - \int_0^a f(a-x)\,dx = 0$

2. $2\int_0^{\pi} xf(\cos^2 x)\,dx - \pi\int_0^{\pi} f(\cos^2 x)\,dx = 0$

Which of the above statements is/are correct?

*(a)* 1 only

*(b)* 2 only

*(c)* Both 1 and 2

*(d)* Neither 1 nor 2

**69.** The solution of the differential equation

$$y\left[2x\sec^2(y^2)\frac{dy}{dx} + y^3\right] = \ln(x^2e^{y^4})$$

is

*(a)* $\sec^3 y^2 = (\ln x)^2 + c$

*(b)* $\sec^3 y^2 = 12(\ln x) + c$

*(c)* $\tan y^2 = (\ln x)^2 + c$

*(d)* None of the above

where $c$ is an arbitrary constant.

**70.** What is

$$\lim_{x\to 0+}\left[\frac{1}{x^2}\right]\ln(\cos x)$$

where $[\cdot]$ denotes the greatest integer function?

*(a)* $-\frac{1}{2}$

*(b)* $-\frac{1}{3}$

*(c)* 0

*(d)* Limit does not exist

**71.** If

$$I_1 = \int e^{2x} \sin\left(\frac{\pi}{3} - x\right) \cos x \, dx$$

$$I_2 = \int e^{2x} \cos\left(\frac{\pi}{3} - x\right) \sin x \, dx$$

then what is $I_1 + I_2$ equal to?

*(a)* $\frac{\sqrt{3}\, e^{2x} \sin x}{2} + c$

*(b)* $\frac{e^{2x} \cos x}{2} + c$

*(c)* $\frac{\sqrt{3}\, e^{2x}}{4} + c$

*(d)* $\frac{e^{2x}}{4} + c$

where $c$ is the constant of integration.

**72.** What is the general solution of the equation

$$\frac{dy}{dx} = \frac{3x - 4y + 1}{4x + 3y + 1} ?$$

*(a)* $(x + 3y)(y - 3x) + 2(y - x) = c$

*(b)* $(x - 3y)(y + 3x) + 2(y - x) = c$

*(c)* $(3y - x)(y + 3x) + 2(y - x) = c$

*(d)* None of the above

where $c$ is an arbitrary constant.

**73.** What is the equation of straight line parallel to the line $3x + 2y + 7 = 0$ and which is such that the sum of its intercepts on the axes is 10?

*(a)* $3x + 2y - 12 = 0$

*(b)* $3x + 2y + 10 = 0$

*(c)* $2x + 3y - 12 = 0$

*(d)* $2x - 3y - 12 = 0$

**74.** A straight line through $P(1, 2)$ is such that its intercept between the axes is bisected at $P$. Its equation is

*(a)* $x + 2y = 4$

*(b)* $2x - y = 4$

*(c)* $2x + y = 4$

*(d)* $x - 2y = 4$

**75.** If the line $y = mx$ meets the lines $x + 2y - 1 = 0$ and $2x - y + 3 = 0$ at the same point, then $m$ is equal to

*(a)* 1

*(b)* 2

*(c)* $-2$

*(d)* $-1$

**76.** Let $A$ and $B$ be two points on $x$-axis and $y$-axis respectively, $O$ being the origin. If the equal sides $OA$ and $OB$, each equal to $a$, are produced to $P$ and $Q$ respectively such that $AP \cdot BQ = OA \cdot OB$, then the line $PQ$ always passes through the fixed point

*(a)* $\left(\frac{a}{4}, \frac{a}{4}\right)$

*(b)* $\left(\frac{a}{3}, \frac{a}{3}\right)$

*(c)* $\left(\frac{a}{2}, \frac{a}{2}\right)$

*(d)* $(a, a)$

**77.** The new position of the point (1, 2) under rotation through an angle of 90° about the origin in anti-clockwise direction is

*(a)* $(-2, 1)$

*(b)* $(2, -1)$

*(c)* $(1, -2)$

*(d)* $(-1, 2)$

**78.** What is the area of the triangle with vertices at (0, 0, 0), (2, 0, 0) and (0, −2, 0)?

*(a)* $\frac{1}{2}$ square unit

*(b)* 1 square unit

*(c)* 2 square units

*(d)* 4 square units

**79.** Consider two circles

$$C_1 \equiv x^2 + y^2 = a^2$$

$$C_2 \equiv (x-\alpha)^2 + (y-\beta)^2 = b^2$$

with $C_2$ lying inside $C_1$. A circle $C$ lying inside $C_1$ touches $C_1$ internally and $C_2$ externally. Then the locus of the centre of the circle $C$ is

*(a)* a circle of radius $a - b$

*(b)* a parabola of semilatus rectum $a + b$

*(c)* an ellipse of major axis $a + b$

*(d)* None of the above

**80.** The shortest distance of a point from the $x$-axis, $y$-axis and $z$-axis respectively are 2, 3, 6. What is the distance of the point from the origin?

*(a)* $\frac{7}{\sqrt{2}}$ *(b)* 7

*(c)* 11 *(d)* $\frac{49}{2}$

For the next **02 (two)** items that follow :

Consider a plane parallel to $x$-axis and passing through the points (0, 1, 3) and (2, 4, 5).

**81.** What are the direction ratios of normal to the plane?

*(a)* $\langle 1, 2, -3 \rangle$

*(b)* $\langle 4, -6, 0 \rangle$

*(c)* $\langle 1, 2, 3 \rangle$

*(d)* None of the above

**82.** What is the equation to the plane?

*(a)* $2y - 3z + 7 = 0$

*(b)* $x + 2y - 3z + 5 = 0$

*(c)* $2y - 3z + 9 = 0$

*(d)* None of the above

For the next **03 (three)** items that follow :

Consider a unit cube.

**83.** What is the perpendicular distance of a corner to the diagonal not passing through that corner?

(a) $\frac{2}{3}$

(b) $\frac{\sqrt{3}}{2}$

(c) $\sqrt{\frac{2}{3}}$

(d) None of the above

**84.** What is the sum of squares of direction cosines of all the four diagonals of the cube?

(a) 1

(b) 2

(c) 4

(d) Cannot be determined as the data is inadequate

**85.** If $\theta$ is the acute angle between any two diagonals of the cube, then what is $\tan^2\theta$ equal to?

(a) 1

(b) 2

(c) 4

(d) None of the above

**86.** If $\vec{a}, \vec{b}$ are two unit vectors inclined at angle $\theta$ such that $\vec{a}+\vec{b}$ is a unit vector, then what is $\theta$ equal to?

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{6}$ (d) $\frac{2\pi}{3}$

**87.** If $\vec{a}$ is a non-zero vector of magnitude $a$, then $m\vec{a}$ is a unit vector if

(a) $m = \pm 1$

(b) $a = \frac{1}{|m|}$

(c) $a = |m|$

(d) $a = m$

For the next **02 (two)** items that follow :

The vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ are of same length and equally inclined to each other. Let $\vec{a} = \hat{i} + \hat{j}$ and $\vec{b} = \hat{j} + \hat{k}$.

**88.** What is the angle between $\vec{b}$ and $\vec{c}$?

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{6}$ (d) $\frac{\pi}{2}$

**89.** What can be the direction ratios of $\vec{c}$?

(a) $\langle 1, 2, -3\rangle$

(b) $\langle -1, 2, -1\rangle$

(c) $\langle -1, 4, -1\rangle$

(d) None of the above

**90.** If $A$ and $B$ are two matrices such that $AB = B$ and $BA = A$, then what is $A(A-1) + B(B-1)$ equal to?

(a) $AB$

(b) $2AB$

(c) Zero matrix

(d) Identity matrix

**91.** In four throws of a fair die, what is the probability of getting a score of more than 4 at least once?

*(a)* $\frac{65}{81}$

*(b)* $\frac{80}{81}$

*(c)* $\frac{7}{9}$

*(d)* None of the above

For the next **03 (three)** items that follow :

A die is rolled so that the probability of face $m$ is proportional to $m$, where $m = 1, 2, 3, 4, 5, 6$.

**92.** What is the proportionality constant?

*(a)* $\frac{1}{6}$ *(b)* $\frac{1}{14}$

*(c)* $\frac{1}{21}$ *(d)* $\frac{1}{36}$

**93.** What is the probability of getting an even number?

*(a)* $\frac{1}{2}$ *(b)* $\frac{1}{7}$

*(c)* $\frac{4}{7}$ *(d)* $\frac{1}{4}$

**94.** What is the probability of getting a multiple of 3?

*(a)* $\frac{3}{7}$

*(b)* $\frac{2}{21}$

*(c)* $\frac{2}{3}$

*(d)* None of the above

**95.** The probability of a shooter hitting a target is $\frac{2}{3}$. What is the minimum number of times that the shooter must fire so that the probability of hitting the target at least once is more than 0·99?

*(a)* 4

*(b)* 5

*(c)* 6

*(d)* None of the above

**96.** Consider the following statements about the random variables $X$ and $Y$ on the same sample space $S$ :

1. $(X + Y)(s) = X(s) + Y(s)$
2. $(XY)(s) = X(s)Y(s)$

Which of the above statements is/are correct?

*(a)* 1 only

*(b)* 2 only

*(c)* Both 1 and 2

*(d)* Neither 1 nor 2

**97.** Consider the following statements :

1. Area under a histogram gives total frequency.
2. Width of the tallest vertical bar of the histogram gives modal class.

Which of the above statements is/are correct?

*(a)* 1 only

*(b)* 2 only

*(c)* Both 1 and 2

*(d)* Neither 1 nor 2

**98.** Consider the following statements related to measure of central tendency of 50 positive numbers :

1. The median is not influenced by extreme values in the set of numbers.
2. The harmonic mean is unreliable if one or more of the numbers is near zero.

Which of the above statements is/are correct?

*(a)* 1 only

*(b)* 2 only

*(c)* Both 1 and 2

*(d)* Neither 1 nor 2

**99.** A fair coin is tossed 6 times; call heads a success. This is a binomial experiment with $n = 6$ and $p = q = \frac{1}{2}$.

What is the probability of getting at least 4 heads?

*(a)* $\frac{1}{2}$

*(b)* 1

*(c)* $\frac{11}{32}$

*(d)* None of the above

**100.** If $A$ and $B$ are two events such that

$$P(A \cup B) = \frac{3}{4},\ P(A \cap B) = \frac{1}{4},\ P(\text{not } A) = \frac{2}{3}$$

then what is $P(B)$ equal to?

*(a)* $\frac{1}{3}$

*(b)* $\frac{2}{3}$

*(c)* $\frac{1}{9}$

*(d)* $\frac{2}{9}$

**SPACE FOR ROUGH WORK**

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